ISSN 2454-5554
An International Peer Reviewed & Referred
INTERNATIONAL JOURNAL
OF EDUCATIONAL RESEARCH STUDIES

SUSTAINABLE MODELS FOR BETTER DESIGN OF FOUNDATION SYSTEMS

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Abstract

Soil–structure interaction is an interdisciplinary field of endeavour which lies at the intersection of soil and structural mechanics; soil and structural dynamics; computational and numerical methods. Its origins trace back to the late 19th century, evolved and matured gradually in the ensuing decades and during the first half of the 20th century, and progressed rapidly in the second half stimulated mainly by the needs of the nuclear power and offshore industries, and simultaneously, by the debut of powerful computers and simulation tools such as finite elements. However, the actual behaviour of a concrete element resting over the soil media is very complicated and it shows a great variety of behaviour when subjected to different conditions because of heterogeneous nature of both concrete and soil. Various constitutive models have been proposed by several researchers to describe different aspects of concrete and soil behaviour in details. But none of these models is capable to completely describe the complex behaviour of this composite-system arsing out by construction of a concrete element over the soil media under all conditions. This paper attempts to present various constitutive models developed by researchers and could be helpful in developing an analytical model to describe the behaviour of foundation systems by considering the soil-structure interaction in a better way thereby, paving a way for formulation of sustainable design methodologies by reducing the embodied energy through a better design.

Key words: *Soils; Constitutive model; Analysis; Soil-structure interaction; Modelling.*

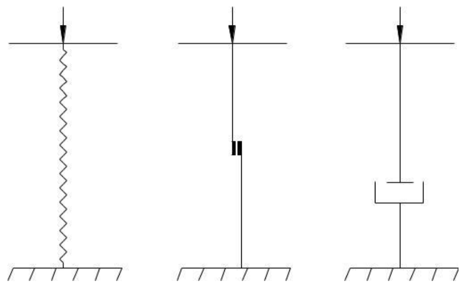
Introduction

Modelling is an art and science to represent the aspect of real behaviourand mathematics provides the frame work to representing it with well defined formulas and rules .if the real behavior is identified from certain formula then it is compared with actual data ,if it don't match then may be revised or rejected DUE to heterogeneity behavior of both the concrete and soil it's the difficult to derive the perfect mathematical formula for their interaction behavior of them . The concrete have low tensile strength results in cracking of concrete which results in non linear behavior of concrete . Other time-independent nonlinearity arise in the RC structural element from the nonlinear action of the individual constituents of reinforced concrete viz: bond slip between rebar and concrete, aggregate interlock of a cracked concrete, and dowel action of rebars. In case of a soil media, modelling a problem become an extremely difficult task due to dependence of the shear and compressive strength of soil on the confining pressure. The tensile strength of soil depends entirely upon the presence of *c*-content in the media. However, the tensile strength of soil is of little use in real application similar to concrete and it is hardly taken into consideration during modelling. Therefore, the behaviour of reinforced concrete element resting over the soil media is influenced by numerous factors constituting the system viz: concrete, embedded rebars and their interaction among themselves, soil media and factors controlling the soil properties. The relationships constituting the system behaviour are known as constitutive relations or model. These are vital part of any numerical computer code which has become very common these days because of rapid advancement of computing tools and hardware. Paper provides a comparative birds' view to various constitutive models developed so far for modeling different aspects of field problems and these can be helpful to analyst in choosing a right one for the soil- structure interaction analysis. While fairly realistic and efficient models of the material properties and the mechanical behaviour of the beam in space can be established by using the Timoshenko or even the Bernoulli–Euler theory; however, the characteristics that represent the mechanical behaviour of the subsoil and its interaction with the beam resting on it are difficult to model. Assuming a linear elastic, homogeneous and isotropic behaviour of the soil, two major classes of soil models can be identified in the literature: (a) continuous medium models, and (b) mechanical models

Constitutive Model

The complexity in the behavior of soil has led the development of many models of soil based on classical theory of elasticity, plasticity and visco-elasticity for the analysis of soil–structure interaction problems. The elastic property, plastic property and viscous property of soil is represented by using a spring, slider and dashpot respectively (*Kok et al. 2009*). The schematic sketches of these standard models are shown in Figure 1. These models can be used to define the load-displacement relationship of material depending upon its physical properties like stiffness, damping/viscous nature and internal friction developed through aggregate interlock, dowel action, particle-to-particle contact pressure, plasticity being induced in the material at higher load level etc.

Load Load Load



Spring Slider Dashpot

Figure 1 Basic Component used to constitute material behavior

Elastic Models

An elastic model is defined to be one for which the stress depends only on the strain and not the history of that strain. When soil or concrete exhibits purely elastic characteristics, elastic models are considered. This generally happens at low stress levels. The simplest type of idealized soil response is to assume the behavior supporting soil medium as a linear elastic continuum. Here the deformations are assumed as linear and reversible (*Kavitha et al.2010*). Two basic classical approaches have been used and reported in literature for a physically close and mathematically simple model to represent soil in the soil structure interaction problem. These basic elastic models are Winkler model and Continuum model (*Datta and Roy 2002*) and are described in following section.

Winkler Model

In Winkler model soil medium is represented by a system of identical but mutually independent-closely spaced-discrete-linearly elastic springs. According to this idealization, beam-supporting soil is modelled as a series of closely spaced, mutually independent, linear elastic vertical springs which, evidently, provide resistance in direct proportion to the deflection of the beam. Fig. 2 shows physical representation of Winkler model of soil structure interface. In the Winkler model, the properties of the soil are described only by one parameter k, which represents the stiffness of the vertical spring. Because of its simple mathematic formulation, this one-parameter model can be easily employed in a variety of problems and gives satisfactory results in many practical situations. However, it is considered as a rather crude approximation of the true mechanical behavior of the soil material, mainly due to its inability to take into account the continuity or cohesion of the soil. This limitation, i.e., the assumption that there is no interaction between adjacent springs, also results in overlooking the influence of the soil on either side of the beam.

Load Foundation

Spring

Rigid layer

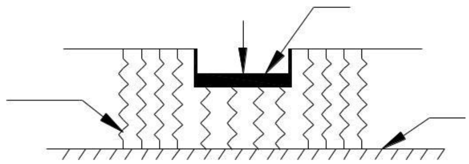


Figure 2 Winkler Model

The pressure deflection relation at any point is given by equation 1.

$$P = k\,w \qquad (1)$$

Where p is the pressure, k is the coefficient of subgrade reaction or subgrade modulus and w is the deflection. The most serious demerit of Winkler model is the one pertaining to the independence of spring. It does not account for the dispersion of load over a gradually increasing influence area with increasing depth and coupling of these vertical springs. This implies that cohesive bond existing among the particles comprising soil medium cannot be considered in this model. The effect of externally applied load therefore gets localized to the point of application only.

Elastic Continuum Model

Soil mass basically constitute of discrete particles compacted by some intergranular forces. The problem commonly dealt in soil mechanics involve boundary distances and loaded areas which are very large compared to the size of individual soil grains. Hence, in effect the body of composed discrete molecules get transformed into a ‘statistical macroscopic equivalent’ amenable to mathematical analysis. Therefore it is reasonable to apply the theory of continuum mechanic for idealizing the soil media. In elastic continuum model the continuous behaviour of soil is idealized as three dimensional continuous elastic solid. In this case the soil surface deflections due to loading will occur under and around the loaded region. Fig. 3 shows typical surface displacement profile of a soil medium subjected to a uniform load ‘P’ of radius ‘a’. The distribution displacements and stresses in such media remain continuous under the action of external force system. In this case some continuous function is assumed to represent the behaviour of soil medium. In continuum idealization soil is assumed to be semi-infinite and isotropic for the sake of simplicity. However the effect of soil layering and anisotropy may be conveniently accounted for in the analysis.

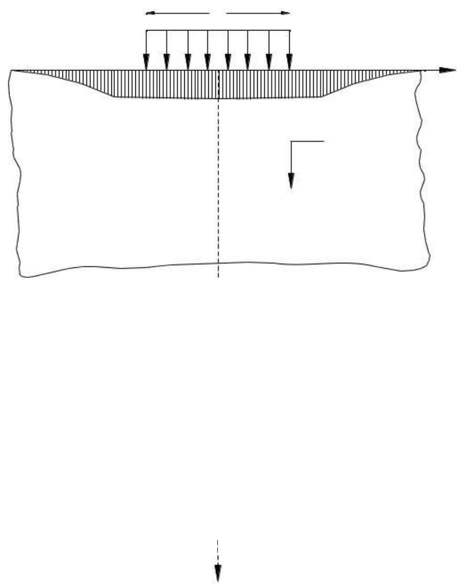


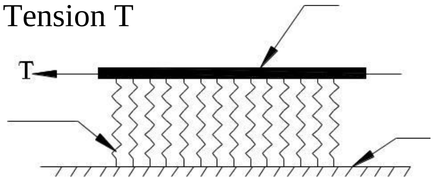
Figure 3 Elastic Continuum Model

This approach provides much more information on the stress and deformations within the soil mass than the Winkler model. However this idealization has a major drawback of inaccuracy in reactions calculated at the peripheries of foundation. It has also been found that in real situation the surface displacements away from loaded region decreased more rapidly than what is predicted for soil by this approach.

Filonenko–Borodich Model

Filonenko–Borodich Model is an improved version of the Wrinkle model. In this model, the first parameter represents the stiffness of the vertical spring, as in the Winkler model, whereas the second parameter is introduced to account for the coupling effect of the linear elastic springs. It is important to note that the interaction enabled by introduction of second parameter also allows the consideration of the influence of the soil on either side of the beam. Despite the introduction of a second parameter, the mathematical formulation of the problem and the corresponding analytical solutions remain relatively simple. In this model, a smooth thin elastic membrane has been used to model the continuity existing between various Wrinkle springs. A uniform tension, T is applied through this elastic membrane at top face of all springs and it is responsible for maintaining the continuity of deformations. Thus, two-parameter models are less restrictive than the Winkler model but not as complicated as the elastic continuum model. Fig.4 shows the physical representation of Filonenko–Borodich Model. The mathematical equation governing this model is given in Eq. (2).

Streched Membrane with Tension T



T

Spring

Rigid layer

Figure 4 Filonenko–Borodich Model

The governing expression of this model is given in equation (2).

$p = kw - T\nabla^2 w$ (For rectangular or circular footing)

$p = kw - T\frac{d^2w}{dx^2}$ (For strip foundation) (2)

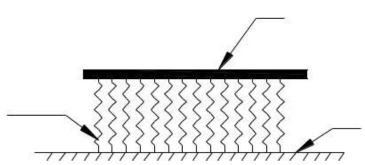
Where, $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ Laplace operator

T = Tensile force in membrane.

Pasternak Model

Pasternak Model is also an improved version of the Wrinkle model. Fig.5 shows the physical representation of Pasternak Model. In this case, a shear layer of unit thickness is combined at top of Winkler springs to represent both compressibility and shear stiffness of the soil or subgrade to model continuity of soil deformations on the surface. The shear interaction between the Winkler spring is characterized by the shear stiffness 'G_p' (=G_sH).

Shear Layer with



Shear modulus G

Spring

Rigid layer

Figure 5 Pasternak Model

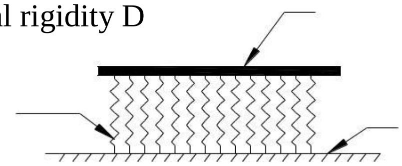
The governing expression can be expressed by equation (3). $p = kw - G_p \frac{d^2w}{dx^2} = kw - G_s H \frac{d^2w}{dx^2}$ (3)

Where, G_sH is the product of the shear stiffness and the thickness of the layer and represents the overall stiffness of the layer.

Hetenyi's Model

Hetenyi's Model is also an improved version of the previous two models. In this model, shown in Fig. 6, the interaction among the discrete springs is accompanied by incorporating an elastic beam or an elastic plate of flexural rigidity EI, which undergoes only flexural deformations upon loading. This model can be used to predict the stress field in raft or any other foundation system and better reflect the true behavior of the system than Wrinkle Model or its modified versions.

Beam or Plate with flexural rigidity D



Spring

Rigid layer

Figure 6 Hetenyi's Model The governing equation of the model is given in equation (4).

(4) $p = kw + D\nabla^4 w$

Where, D = Flexural rigidity of the elastic-plate $= \frac{E_p h_p^3}{12(1-\mu_p^2)}$

p = Pressure at the interface of the plate and the spring

And E_p , μ_p are the Young's modulus and Poisson's ratio of plate material respectively. h_p = Thickness of the plate and operator, ∇^4 is given below:

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2}$$

Kerr Model

Kerr Model is a three parameter model, which consists of two layers of elastic springs interconnected by an elastic shear layer. The model is shown in the Fig. 7. One of the basic features of Kerr Model is the flexibility and convenience that they offer in the determination of the level of *continuity* of the vertical displacements at the boundaries between the loaded and the unloaded surfaces of the soil.

This feature renders them capable of distributing stresses correctly, whether the soil is cohesive or non-cohesive in nature.

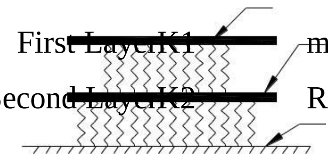


Figure 7 Kerr Model

The governing differential equation is expressed in equation (5).

$$\left(1+\frac{k_1}{k_2}\right)p = \frac{G}{k_1}\nabla^2 p + k_2 w - G\nabla^2 w$$

$$\left(1+\frac{k_1}{k_2}\right)p = \frac{GH}{k_1}\frac{d^2p}{dx^2} + kw - GH\frac{d^2w}{dx^2}$$

(5)

Where k_1 is the spring constant of first layer and k_2 is the spring constant of the second layer, w is the deflection of first layer and GH = shear stiffness of the shear layer

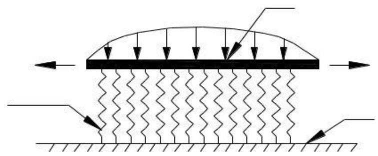
Advantages of Kerr's model (*Madhav 1998*)

- Contact pressure response does not include concentrated reactions.
- An additional parameter is available for filling the theoretical model with the experimental results.
- In case of a layer of finite thickness, an additional boundary condition on shear layer deflection is available to simulate the restraint of foundation layer.

Beam Column Analogy Model

The classical problem of beams on elastic foundation can be solved using a new subgrade model (*Horvath 1993*). The schematic diagram of the model is given in Fig. 8 and it represents a combined soil subgrade plus structural element in contact with the subgrade (e.g., a mat foundation). The structural portion of the model is a conventional flexural element. The subgrade portion of the model is Pasternak's hypothesis, which is fundamentally more accurate than the commonly used Winkler hypothesis. The model can be visualized as a spring-supported beam- column under constant axial tension. The column tension and springs represent the subgrade effects. This beam-column analogy is useful in practice because it allows more accurate modeling of soil-structure interaction within the capabilities of existing structural analysis computer software.

Applied force



Spring

Rigid layer

Figure 8 Beam Column Analogy Model The governing differential equation is given in equation (6).

$$E_b I_b \frac{d^4 w(x)}{dx^4} - C_{p_2} \frac{d^2 w(x)}{dx^2} + C_{p_1} w(x) = q(x)$$

(6)

Where, E_b and I_b is the flexural stiffness of the beam w(x) is the beam settlement q(x) is the applied load

C_{p1} and C_{p2} are constants

For an isotropic, homogeneous layer underlain by a rigid base, $C_{p_1} = \frac{E}{H}$ and $C_{p_2} = \frac{GH}{2}$ where E & G are Young's and shear modulus of assumed shear base.

New Continuous Winker Model

To model the continuity in soil medium, generally some structural element is introduced. But in this model springs are intermeshed so that interconnection is automatically achieved (*Kurian 2001*). Fig. 9 shows the schematic representation of model.

Foundation

Load

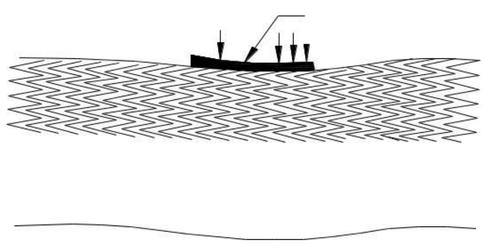


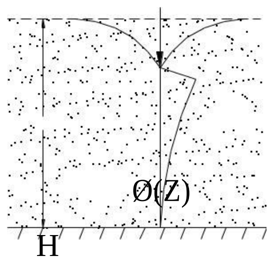
Figure 9 New Continuous Winker Model

Interconnection among Winkler spring connected to foundation beam or plate is achieved by some other spring by virtue of their axial stiffness not directly attached to the foundation. This model is able to account for the effect of soil outside the boundaries of the structure in the modeling.

Vlasov Model (Improved version of continuum model)

The model of soil response proposed by *Vlasov (1949)* is derived by introducing displacement constraints that simplify the basic equations of linear theory of elasticity for an isotropic continuum and using the variational approach. This formulation provides a rigorous theoretical basis for the present form of the vertical deformation profiles, which have previously been assumed arbitrarily. This model is shown in Fig. 10.

P



Es. νs

Z

Figure 10 Vlasov Model

The state of strain in the subgrade is assumed to be such that the horizontal displacements are zero and the vertical displacements can be expressed through an equation (7).

$$w(x,z) = w(x).h(z) \qquad (7)$$

Where, the function h(z) prescribes the variation of displacements with depth z from the surface. The linear and exponential variations for thin and thick deposits are

$$h(z) = (1 - \eta)$$

$$h(z) = \frac{\sinh\left[\frac{\gamma(H - z)}{L}\right]}{\sinh\left[\frac{\gamma H}{L}\right]}$$

Where, $\eta = \frac{z}{H}$ and γ & L are constants.

The governing equation is

$$p = kw - 2t\frac{d^2w}{dx^2}$$

where,

$$k = \frac{E_0}{H(1-\vartheta_0^2)}, t = \frac{E_0 H}{12(1+\vartheta_0)}, E_0 = \frac{E_s}{1-\vartheta_s^2} \text{ and } n_0 = \frac{\vartheta_s}{1-\vartheta_s}$$

Reissner Model

Reissner (1958) proposed a model by introducing constraints on displacements and stresses that simply the basic equation for a linear isotropic continuum. This has been shown in Fig. 12. The in-plane (x-y plane) stresses $\sigma_x = \sigma_y = \tau_{xy} = 0$ throughout the depth 'H' of the subgrade and the displacement components u, v and w in the x, y and z direction respectively satisfy the conditions u = v = w = 0 on z = H and u= v = 0 on z = 0. The response function of the Reissner model is given by an expression shown in equation (8).

$$c_1 w - c_2 \frac{d^2 w}{dx^2} = p - \frac{c_2}{4c_1}\frac{d^2 p}{dx^2} \quad (8)$$

Where, w is the displacement and p is a distributed lateral load acting on the foundation surface

$$c_1 = \frac{E_s}{H}, \; c_2 = \frac{HG_s}{3}$$

E_s and G_s are deformation and shear moduli of the subgrade and H is the thickness of the foundation layer.

Elastic Plastic Model

In soil structure interaction analysis, non-linear behavior of soil mass is often modeled in the form of an elastoplastic element. In this case, deformations occur linearly and proportional to the applied stress only up to certain stress level and afterwards, it start following a liner path. This behavior may be represented by an ideal reversible spring.

Mohr-Coulomb Model

It is an elastic –perfectly plastic model. The set of parameters adopted to represent the model are; young's modulus, poisson's ratio, friction angle, cohesion and dilatancy angle (*Wai and Atef 1982*). For each sub layer a linear variation of the young's modulus has been assumed. As the model does not allow the change of the soil stiffness within the strain level, a reduced static stiffness has been adopted during the preliminary construction stage. Mohr- Coulomb model is a simple model applicable to three dimensional stresses with only two strength parameters to describe the plastic behavior. It is indicated in literature that by means of triaxial test, stress combinations causing failure in real soil samples agrees quite well with the hexagonal shape of failure contour. This model is applicable to analyze the stability of dams, embankments and shallow foundations.

St. Venant's Model

In St. Venant's model, an elastic element (Hookean spring) is connected in series with a plastic element as shown in Fig. 12. Use of such a single element generally shows an abrupt transition from elastic to plastic state. The use of large number St. Venant's units in parallel represents the elasto-plastic behavior of soil more accurately and helps in simulating the gradual transition of soil strain from elastic to plastic zone.

Figure 12 St. Venant's Model

The following expression may be used in terms of strain modulai for elastic and plastic strains respectively

$\varepsilon_{ep} = M_e\, \sigma + M_p \log[\sigma/(\sigma_u\text{-}\sigma)]$

Where, M_e and M_p are elastic and plastic strain modulus of soil respectively and σu is the ultimate load. Conceptually the above mechanical model may appear to be useful enough, but the problem occurs in the proper adjustment of such springs at the base of structure.

Viscoelastic model

The real deformation characteristics of fine grained soil media under the application of any load are always time dependent and to some extend depending on the permeability of soil media. Loading applied to a saturated layer of clay, at the first instance, causes an increase in pressure in the pore water of soil. With time pore water pressure will dissipate resulting in progressive increase of effective stress in soil skeleton. This leads to time dependent settlement of foundation. There are numerous instances of rheological processes in foundation leading to large and non-uniform settlement. The mechanical model represents the rheological properties of soil skeleton by a combination of elastic, viscous and plastic elements. These models are generally formed by a combination of springs and dashpots in series or in parallel. Various models are available to describe the rheological properties of clayey soil.

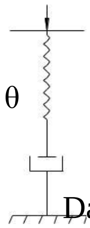
Maxwell's Model

It is the most general form of the linear model for viscoelasticity. In this model a spring and dashpot are connected in series and subjected to load as shown in Fig. 13.

Load P

Spring k

Dashpot coefficent θ



Dashpot

Load P

Dashpot coefficent θ

Spring k

P

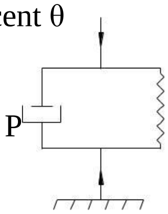


Figure13 Maxwell's Model   Figure 14 Kelvin's Model

For better results, a number of such arrangements in parallel may be used. Since stress σ is common to both the elements, total strain can be calculated from following equation.

$\varepsilon_{total} = \varepsilon_s + \varepsilon_D = \sigma/K + (\sigma/\eta)t$

Where ε_s and ε_D are strain in spring and dashpot and η viscosity.

Kelvin Model

It is a viscoelastic model having the properties of elasticity and viscosity. It can be represented by a purely viscous damper and purely elastic spring connected in parallel as shown in Fig. 14. Since the two components of the model are arranged in parallel, the strain in each component is identical

$\varepsilon total = \varepsilon s = \varepsilon D$

Similarly the total stress will be the sum of the stress in each component

$\sigma total = \sigma D + \sigma S$

From these equations stress σ, strain ε and their rate of change with respect to time, t are governed by the following equation

$\sigma(t) = E\varepsilon(t) + \eta\{d\ \varepsilon(t)/dt\}$

Where, E is the modulus of elasticity.

Again the strain is obtained as $\varepsilon(t) = \{\sigma(t)/k\}[1- e(-kt/\ \eta)]$

Concluding Remarks

The review of various constitutive soil models as applied in soil-structure interaction analysis leads to following broad conclusion.

1. For all practical purpose Winkler hypothesis idealization yields reasonable results.
2. Modeling of inelastic materials phenomenon in the triaxial range is still limited to very restrictive test conditions. At present, there is no unified formulation which describes the nonlinear

material response in the pre and post failure regime. However, in order to estimate the design force accurately, the knowledge of true constitutive relationship is necessary.

3. Soil structure interaction of clayey soil having low permeability depends on time dependent behavior under sustained loading and critical condition may occur at any time during the process. Under such circumstances the crucial input for design can be obtained only if the soil is modeled as viscoelastic medium.

4. The strengths and limitations discussed in the paper will help in selecting the suitable model based on the requirement of the problem.

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